The tensile fracture stress of capsule-shaped tablets

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A recent paper by Gold et al (1980) has suggested that the determination of the mechanical strength of capsule-shaped tablets can be undertaken by what would be described, in the field of material science, as a flexure test. This type of test has been proposed for conventional tablets by David & Augsburger (1974). A feature of such a test is that under the correct conditions of loading, the specimen will be subjected to a pure longitudinal tensile stress along a line on the surface opposite to that on which the load is applied; see Fig. 1. The loading and support point details are important in this form of test; correct conditions are readily recognizable by the occurrence of a clean break across or near to the loading line of the specimen. The purpose of the present communication is to show how it is possible to determine the tensile strength of capsule-shaped tablets from this type of flexure test.

In general, for a beam specimen subjected to bending, the tensile fracture stress σ_f can be calculated from the following expression

$$\sigma_f = \frac{My_{max}}{I} \quad \ldots \quad (1)$$

where M is the bending moment at fracture, y_{max} is the normal distance from the neutral axis to the furthermost point in the part of the cross-section which is under tension, and I is the second moment of area of the cross-section.

Rectangular cross-section. For a beam of rectangular cross-section, width b and depth (i.e. thickness) d,

$$I = \frac{bd^3}{12} \quad \ldots \quad (2)$$

and

$$y_{max} = \frac{d}{2} \quad \ldots \quad (3)$$

For the symmetrical three-point bending configuration shown in Fig. 1, the maximum bending moment for such a specimen is

$$M = \frac{Wl}{4} \quad \ldots \quad (4)$$

where W is the fracture load, and l is the distance between the supports. (The maximum bending moment expression used by David & Augsburger 1974, is correct but the argument leading to their final equation for the tensile strength and the final equation itself are invalid.) Thus at failure, from the above equations:

$$\sigma_f = \frac{Wl}{4}\frac{d}{2}\frac{12}{bd^3} \quad \ldots \quad (5)$$

$$= \frac{3Wl}{2bd^2} \quad \ldots \quad (6)$$

Capsule cross-section. The section of a capsule-shaped tablet is now considered; the definitive dimensions are given in Fig. 2. The tensile fracture stress σ_f can be determined as before from equation (1), but, whilst the bending moment expression (eqn 4) remains unchanged, the values of I, the second moment of area of the cross-section, and y_{max} have to be redetermined.

The first step is to obtain the second moment of area of one curved segment (the shaded area in Fig. 2) about a parallel axis though its centre of area, I_{11}—see Fig. 3. To do this it is necessary to calculate the angle α, subtended by the curved portion of the surface at its centre, in terms of the dimensions given in Fig. 2. If the radius of the curved surface of the segment is denoted by *r*, then

$$\tan \alpha = \frac{b}{2(r - a)} \quad \ldots \quad (7)$$

Also, from the 'intersecting chords' theorem,

$$\frac{b^2}{4} = (2r - a)a \quad \ldots \quad (8)$$

from which

$$r = \frac{1}{2}\left(\frac{b^2}{4a} + a\right) \quad \ldots \quad (9)$$

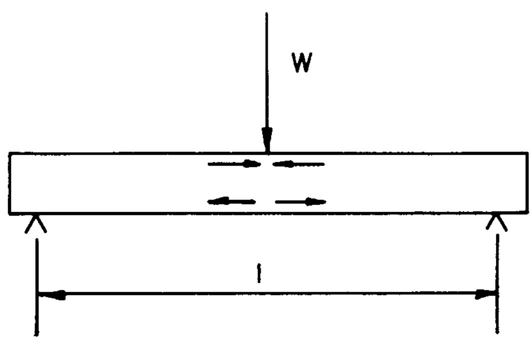

FIG. 1. Flexure test: loading and support.

† Correspondence.

It follows from equations (7) and (9) that

$$\tan \alpha = \frac{2ab}{b^2 - 4a^2} \quad .. \quad .. \quad (10)$$

Using equation (10) the value of α is readily determined from the tablet dimensions. For normal tablet dimensions it is likely that the value of α will be greater than 45°. For sections in which $\alpha > 45°$, Roark & Young (1975) give the following formula for I_{11}, the second moment of area about an axis through the centre of area and parallel to the flat edge of the segment:

$$I_{11} = \frac{r^4}{4}\left[\alpha - \sin\alpha\cos\alpha + 2\sin^3\alpha\cos\alpha - \frac{16\sin^6\alpha}{9(\alpha - \sin\alpha\cos\alpha)}\right] \quad (11)$$

The numerical value of I_{11} can therefore be derived from equations (9), (10) and (11). The magnitude of the distance y_{1b}, the normal distance of the centre of area of the segment from the flat edge (see Fig. 3), is given by the expression (Roark & Young 1975):

$$y_{1b} = r\left[\frac{2\sin^3\alpha}{3(\alpha - \sin\alpha\cos\alpha)} - \cos\alpha\right] \quad .. \quad .. \quad (12)$$

from which the numerical value follows. The area of the curved segment A (Roark & Young 1975) is given by

$$A = r^2(\alpha - \sin\alpha\cos\alpha) \quad .. \quad .. \quad .. \quad (13)$$

Using the 'parallel axis' theorem (Timoshenko & Young 1962) the second moment of area of the full section about the axis AA through the centre of area now follows as:

$$I_{AA} = 2\left[I_{11} + A\left(y_{1b} + \frac{d}{2}\right)^2\right] + \frac{bd^3}{12} \quad .. \quad (14)$$

Nomenclature

a	height of curved segment
A	area of curved segment
b	width of beam or tablet
d	depth or thickness of beam or a tablet dimension (see Fig. 2)
I	second moment of area of cross-section
I_{AA}	second moment of area of cross-section of capsule-shaped tablet
I_{11}	second moment of area of cross-section of curved segment of capsule-shaped tablet
l	distance between support points of beam or tablet
M	bending moment
r	radius of curved surface of the capsule-shaped tablet
W	fracture load in three-point bending
y_{max}	normal distance from the neutral axis to the furthest point in the part of the cross-section which is in tension
y_{1b}	normal distance of the centre of area of segment from the flat edge
α	Half-angle subtended by segment formed by curved surface of tablet
σ_f	tensile fracture stress

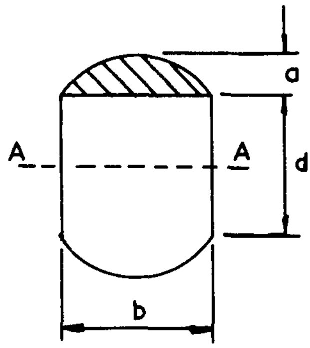

FIG. 2. Cross-section of capsule-shaped tablet.

This, the required quantity, can be evaluated from the tablet dimensions using equations (11), (12) and (13).

The quantity y_{max} (equation (1)) for this case is given by

$$y_{max} = \frac{d}{2} + a \quad .. \quad .. \quad (15)$$

The final stage in the process is to substitute into equation (1) to give the tensile fracture stress in terms of the fracture load and the tablet dimensions:

$$\sigma_f = \frac{Wl}{4}\frac{(d/2 + a)}{I_{AA}} \quad .. \quad .. \quad (16)$$

Thus knowing tablet dimensions and the fracture load it is possible to arrive at a fundamental value for the tensile fracture stress of capsule-shaped tablets from a flexure test experiment. (It has been assumed that the applied load W is distributed across the width of the tablet and contact effects at the loading point have been ignored.)

If a typical capsule-shaped tablet is considered, it is interesting to observe in equation (14) that y_{1b} may be

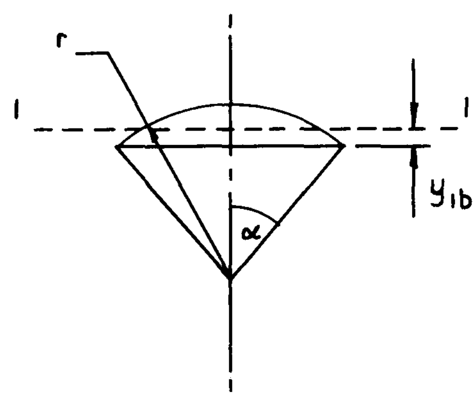

FIG. 3. Details of segment.

negligible compared with d/2 and I_{11} may be negligible compared with $Ad^2/4$. It would follow as an approximation that

$$I_{AA} \simeq \frac{Ad^2}{2} + \frac{bd^3}{12}$$

and for practical purposes this could be used as an effective approximation, in the form:

$$\sigma_t \simeq \frac{3}{2}\frac{Wl}{d^2}\left[\frac{d + 2a}{6A + bd}\right] \quad \ldots \quad (17)$$

This simplification avoids the calculations of I_{11} and y_{1b} and provides a relatively simple approximate solution to what at first might appear to be a difficult problem. (It should be mentioned that the corresponding approximate tensile fracture stress (eqn 17) would err on the high side of the true value.)

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Punch tip geometry effects on powder compression

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Many workers have studied the compression process and found it to exhibit distinct phases (Seelig & Wulff 1946; Train 1956; Marshall 1970). During the major densification stage corresponding to particle deformation and/or fracture and recombination a linear relationship has been shown to exist between tablet relative volume and log compressional force (Walker 1923), and between tablet porosity and the reciprocal of log compressional force (Higuchi et al 1954). All these studies were carried out using flat faced punches. Aulton et al (1975) however, demonstrated a variation in surface hardness of tablets with alteration in punch face geometry.

To investigate the effect of punch shape on previously found relationships tablets were prepared using four different shaped punch faces namely flat faced, bevelled, concave and deep concave. The tablets were prepared from Avicel PH101 (F.M.C. Corpn. Marcus Hook, U.S.A.). This powder was granulated before compression using Avicel: water ratio of 5:2, mixed for 10 min in a Z blade mixer (Baker Perkins, Peterborough) passed through a 210 μm mesh on an oscillating granulator (Manesty Machines, Liverpool), dried to a final moisture content of 5% w/w and repassed through a 210 μm mesh. The moisture content was determined using an Ohaus moisture balance. The prepared granules were mechanically sieved and the 105–108 μm fractions retained and stored in sealed jars until compressed.

Tablets were prepared using a 30 m.m i.d. punch and die set and a hydraulic press, at five levels of compressional force (F_A) 4·9 kN; 9·8 kN; 24·5 kN; 49·0 kN and 98·1 kN.

The thickness of each tablet was measured, immediately after ejection from the die, using a micrometer gauge, fixed rigidly on a metal stand, and capable of measuring $\pm$ 0·01 mm. The average height of four tablets was found and used in calculations of tablet relative volume (R.V) and tablet porosity (ϵ), these values being calculated assuming the punch tip profile to be identical with that of the tablet.

From Tables 1 and 2 it can be seen that for the flat faced punch the previously deduced relationships, namely R.V$\alpha\log_{10}F_A$ (Walker 1923) and reciprocal of $\log_{10} F_A\alpha\epsilon$ (Higuchi et al 1954) are also valid in this case. When considering the shaped tablets the same relation-

Table 1. Relationship between tablet relative volume and compressional force for all punch tip geometries.

Compression force (kN)	Relative volume (R.V.)					
			Concave		Deep concave	
	Flat	Bevelled	(a)	(b)	(c)	(d)
4·9	2·06	2·08	2·48	2·51	2·91	2·56
9·8	1·76	1·79	2·219	2·10	2·61	2·08
24·5	1·39	1·35	1·60	1·68	2·11	1·68
49·0	1·23	1·25	1·54	1·49	1·74	1·57
98·1	1·07	1·11	1·30	1·28	1·54	1·42
Correlation coefficient $\log_{10}F_A$ vs R.V.	—1·0130	—0·999	—0·999	—1·011	—1·019	—0·988